Unit Title and Subject Area: Pythagorean Theorem in Geometry

Grade Level: 11

Goals:

Grade Level Content Expectations:

Michigan High School Content Expectations (HSCE):

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF
Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem in multi-step problems.

NETS•S (2007) Performance Indicators for The Grade Level

Profile for Technology (ICT) Literate Students
Grades 9–12 (Ages 14–18)

4. Employ curriculum-specific simulations to practice critical-thinking processes. (1.Creativity and Innovation and 4.Critical Thinking, Problem Solving, and Decision Making)

6. Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning, and career needs. (4.Critical Thinking, Problem Solving, and Decision Making; 5. Digital Citizenship; and 6. Technology Operations and Concepts)

Learning Objectives:

1. Knows Pythagorean Theorem. (Knowledge)
   1.1 Describes a right triangle.

Identifies the legs and hypotenuse of a right triangle.
Defines Pythagorean Theorem.

2. Understands the concept of Pythagorean Theorem. (Comprehension)
   2.1 Interprets Pythagorean Theorem.

Generalizes a pattern from Pythagorean Theorem.

3. Applies the concept of Pythagorean Theorem. (Application)
   3.1 Relates the pattern of Pythagorean Theorem with Geometer's Sketchpad.
4. Analyzes the uses of the Pythagorean Theorem. (Analysis)
   4.1 Illustrates a conclusion on Pythagorean Theorem with informal proof using Geometer's Sketchpad.

5. Integrates prior reasoning experience to Pythagorean Theorem. (Synthesis)
   5.1 Writes an algebraic representation of the visualized Pythagorean Theorem.

6. Knows the uses of Pythagorean Theorem. (Knowledge)
   6.1 Defines Pythagorean Triple.
   6.2 Identifies the use of Pythagorean Theorem in measurement problems.

7. Understands the uses of Pythagorean Theorem. (Comprehension)
   7.1 Interprets the use of Pythagorean Theorem in a word problem.
   7.2 Translates a verbal model into a Pythagorean Theorem model.

8. Applies the uses of Pythagorean Theorem. (Application)
   8.1 Computes Pythagorean Triples.
   8.2 Computes for the length of a hypotenuse with Pythagorean Theorem.
   8.3 Computes for the length of a leg with Pythagorean Theorem.
   8.4 Computes for the area with Pythagorean Theorem.

9. Analyzes the uses of Pythagorean Theorem with Geometer's Sketchpad. (Analysis)
   9.1 Illustrates Pythagorean Triples with Geometer's Sketchpad.
   9.2 Demonstrates finding side lengths using Pythagorean Theorem with Geometer's Sketchpad.
   9.3 Shows finding the area using Pythagorean Theorem with Geometer's Sketchpad.

10. Integrates the learning on the uses of Pythagorean Theorem. (Synthesis)
    10.1 Creates a Pythagorean Theorem problem.

11. Knows the rules pertaining to the formal proof of Pythagorean Theorem. (Knowledge)
    11.1 Defines Perpendicular Postulate.
    11.2 Defines Geometric Mean Theorem.
    11.3 Defines Addition Property of Equality.
    11.4 Defines Distributive Property.
    11.5 Defines Segment Addition Postulate.
    11.6 Defines Substitution Property of Equality.

12. Understands the concept of the rules pertaining to the formal proof of Pythagorean Theorem. (Comprehension)
    12.1 Explains Perpendicular Postulate proof based on Internet research.
    12.2 Explains Geometric Mean Theorem proof based on Internet research.
    12.3 Explains Addition Property of Equality proof based on Internet research.
    12.4 Explains Distributive Property proof based on Internet research.
    12.5 Explains Segment Addition Postulate proof based on Internet research.
    12.6 Explains Substitution Property of Equality

13. Analyzes the concept of the rules pertaining to the formal proof of Pythagorean Theorem. (Analysis)
    13.1 Relates the rules to make the formal proof of Pythagorean Theorem.

14. Judges the credibility of a website and the validity of the mathematical proof from the website for the Pythagorean Theorem homework. (Evaluation)
    14.1 Appraises the credentials of the web source and the validity of the mathematical proof on a rule from that web source.

Characteristics of the Students: At this juncture, students should have already been exposed to axioms, postulates, properties, and theorems in addition to having made formal proofs at the beginning of the year. This is intended for students who have by now acquired experiences in geometric principles of reasoning and proof. Thus, it is offered in the last quarter of the year. The students, moreover, finished Algebra 1 and Algebra 2 in Grades 9 and 10, respectively. If digital technology exists, the students are already experienced with the application. This unit lesson plan, however, caters to a body of students with multiple abilities. The instruction is evidently differentiated from its content, interfaces, and activities, including the techniques of incorporating technology.

Instructional Procedures:

Anticipatory Set

Compound Activities (2-3 per session)

Jigsaw Grouping with Multiple Ability Members

Similar Ability Grouping

Differentiated Content and Process

Blend of Lecture, Group Work, Class Discussion, Student Presentations

Mixed Metacognition, Pen-and-Paper Method, and Technology Use

Progressive Learning with Repetition of Concept

PowerPoint Lectures and Gaming

Use of Exit Slips

Worksheet Completion

Materials, Resources, and Technology

Paper, Pens, and Pencils

Calculators

Whiteboard

Eraser

Internet

Geometer's Sketchpad

Printer

PowerPoint Lectures (Informal Proof, Uses, Formal Proof of Pythagorean Theorem)

Pythagoras Square Worksheet

R. O'Donnell Worksheet

Summary Sheet of Student Responses on the Uses of Pythagorean Theorem

Color-coded Worksheets (red, yellow, blue, green)

Homework Handout on Internet Research

Formal Proof Worksheet

Jeopardy Game

Candy Bars for Reward

Assessment Strategies

Formative assessment comprise of participation logs, objective evaluations on computational work, alternative appraisals such as rubrics for worksheets requiring explanations and step by step procedures as well as research work. Summative assessment is based on the unit portfolio of the students' accomplishments in class as reflected on the overall standing in participation, worksheet completion, and presentation grades.

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Lesson Plans

Lesson Plan 1
Informal Proof: Drawing out the Conclusions for Pythagorean Theorem

Lesson Plan 2
Applying Pythagorean Theorem

Lesson Plan 3
Formal Proof of the Pythagorean Theorem

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LESSON PLAN 1

Lesson Title: Informal Proof: Drawing out the Conclusions for the Pythagorean Theorem

Lesson Author: Irma Crespo

Grade Level/Subject Area: Grade 11/Geometry

Time Alloted for Lesson: 45 minutes/One Class Meeting

Short Description of Lesson:
The students learn about Pythagoras, Pythagorean Theorem, informal proofs, and implementing informal proofs through visualizations using Geometer's Sketchpad.

Classroom Layout and Grouping of Students: The set up varies from teacher centered on lectures and modeling, student to student with group work, student to class with presentations, and student personal workstations for individual practice. Grouping is based on seating arrangements.

Grade Level Content Expectations:

Michigan High School Content Expectations (HSCE):

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF
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G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem in multi-step problems.

NETS•S (2007) **Performance Indicators for The Grade Level**

Profile for Technology (ICT) Literate Students
Grades 9–12 (Ages 14–18)

4. Employ curriculum-specific simulations to practice critical-thinking processes. (1.Creativity and Innovation and 4.Critical Thinking, Problem Solving, and Decision Making)

6. Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning, and career needs. (4.Critical Thinking, Problem Solving, and Decision Making, 5. Digital Citizenship, and 6. Technology Operations and Concepts)

Instructional Objectives

6. Knows Pythagorean Theorem. (Knowledge)
   1.1 Describes a right triangle.

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Defines Pythagorean Theorem.

7. Understands the concept of Pythagorean Theorem. (Comprehension)
   2.1 Interprets Pythagorean Theorem.

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   3.1 Relates the pattern of Pythagorean Theorem with Geometer's Sketchpad.

9. Analyzes the uses of the Pythagorean Theorem. (Analysis)
   4.1 Illustrates a conclusion on Pythagorean Theorem with informal proof using Geometer's Sketchpad.

10. Integrates prior reasoning experience to Pythagorean Theorem. (Synthesis)
    5.1 Writes an algebraic representation of the visualized Pythagorean Theorem.
    5.2 Incorporates the lessons to new but relevant conditions.

Materials, Resources, and Technology: PowerPoint presentation, R. O'Donnell worksheet, Geometer's Sketchpad, The Pythagoras Square worksheet, pens, paper, pencils, whiteboard, eraser

Students' Present Level of Performance and Skills: At this juncture, students should have already been exposed to axioms, postulates, properties, and theorems in addition to having made formal proofs at the beginning of the year. This is intended for students who have by now acquired experiences in geometric principles of reasoning and proof. Thus, it is offered in the last quarter of the year. The students, moreover, finished Algebra 1 and Algebra 2 in Grades 9 and 10, respectively. If digital technology exists, the students are already experienced with the application.

Instructional Procedures:

I. Anticipatory Set:
To prepare the students for the actual lesson on Pythagorean Theorem and to determine their prior experiences about this, The Pythagorean Square worksheet is distributed for them to write down their observations and to discuss these interpretations in class. The teacher jots the student responses on the whiteboard during discussions to encourage mathematical discourse. After the talk, the teacher advises the students to set aside the worksheet for an end of class task.

II. Learning Activity:
The teacher opens up PowerPoint and indicates that the previous endeavor was to get them ready for learning Pythagorean Theorem.

1. The teacher presents a brief lecture entitled Meet Pythagoras and His Theorem. A historical overview, definition, and description of the theorem.

2. The teacher explains informal proof with the section called A Casual Inquiry.

3. The teacher distributes the R. O'Donnell worksheet to complete with Geometer's Sketchpad, which at this point, the students have been already exposed to from past lessons.

4. Students are told to read the whole worksheet quietly so they can get acquainted with the task.

5. When the reading is over, the teacher models an outline that gives the main points required by the R. O'Donnell worksheet.

6. The teacher, then, asks if there are any questions or further clarifications needed.

7. The students work on the informal proof following the directions on the R. O'Donnell worksheet.

8. When the time is up, the teacher wraps up by inquiring how the students feel about the undertaking and what they found out after performing the informal proof.

9. The printed (Sketchpad) worksheets are submitted for grading.

III. Closure:
To sum up the hour's discussions, bring a conclusion to the lesson, and help students organize the learning that took place, the students bring out their Pythagoras Square worksheets.

The teacher groups students into pairs to create an algebraic representation of the image on the worksheet following the Pythagorean equation and explain the connection between the equation and the picture. The students are advised to write the answers on their own worksheets because these will be graded individually. However, they also have to jot down the name of their partners on the worksheet.

After the deliberation among group members, the teacher randomly calls groups to expound on their equations.

The Pythagoras worksheets are submitted for grading.

Supplemental Activity: Extensions and Remediation:
For exit slip, the students write a sentence on how they think Pythagorean Theorem can be used. (It is a higher order learning activity that requires synthesis.)

Adaptations for Special Learners:
The R. O'Donnell worksheet is simplified enough for all students to be able to follow the directions since each step has a corresponding visual for familiarity and recognition. Because this is a class that has exposure to the software, some may skip perusing through the directions and go straight to working on the main points of the activity. Thus, the detailed processes are for students who need more guidance. If this is not enough, a one-to-one mentoring takes place. When a situation compels it, procedures are broken down into segments for these students to perform one part at a time. This is the reason the PowerPoint is shown to sum up each core section.

Assessment:

- Students will be assessed during class discussions for participation points recorded in the participation log. (Specific Objective 1: Knowledge)

- Students will be graded for the informal proof they made with the Geometer's Sketchpad (Specific Objectives 2, 3 and 4: Comprehension, Application, and Analysis)
- Students will be scored for individual work on the Pythagoras Square. (Specific Objectives 3 and 5: Application and Synthesis)
- Students will be graded with a rubric for the group work part of the Pythagoras Square. (Specific Objectives 3 and 5: Application and Synthesis)
- Students will earn extra credit points towards a low-scoring homework for completing the exit slip properly. (Specific Objective 5: Synthesis)

Student Products:
Anticipatory set and closure: The Pythagoras Square worksheet
Learning Activity: The Geometer's Sketchpad on Pythagorean Theorem Informal Proof
Supplemental or Extended Activity: The exit slip statement.

TPACK Analysis

I. TPACK Components

Content (C): Informal Proof of Pythagorean Theorem

Pedagogy (P): Visualization, PowerPoint lecture

Technology (T): Geometer's Sketchpad

Content Knowledge (CK):
Pythagorean Theorem is one of the well-known theorems in geometry. Although it emanated from pure mathematics, today's perspectives extend its concept on applied mathematics. Thus, putting a scheme pristinely made for math pleasure (math for the sake of math) into functional use. There are more than one hundred proofs on Pythagorean Theorem, which accounts for its popularity. It is essential for students to learn it as a stepping-stone towards Trigonometry as well as an extension of their geometric skills. There are numerous resources available to study, learn, and teach Pythagorean Theorem. In this lesson plan, content is emphasized on its informal proof where students derive a common sense way of demonstrating the theorem without the rigidity of rules.

Pedagogy Knowledge (PK):
Abstract thinking requires imagination. To concretize Pythagorean Theorem necessitates visualization. In eleventh grade, Steinberg's description of adolescents as "seeing varied possibilities, imagining abstract ideas, thinking about one's own thoughts, viewing matters in multiple dimensions, and possessing a relative outlook" are taken into account in the instruction. By providing them the opportunity to draw generalizations from pictures, moving or static, the students play on various scenarios with visual manipulation and connect fragments of numerous views into an acceptable, non-absolute conclusion, which they leave open for their own contention. To satisfy the non-linear growth, the classroom activities involve various interfaces: teacher to student in PowerPoint lectures, student to teacher through inquiries, student to student in group tasks, student to class with presentations, and interactivity between teacher, student, and technology in class discussions and individual practice. The latter is the interaction of the student with the teacher as a mentor in hands on use of the devices. Moreover, the lesson plan combines metacognition, pen-and-paper, and technology activities. Of course, these become routine structure the more they get familiar with this type of learning

environment. In terms of evaluation, every task has an artifact assessed not only to gauge the quality of the students' learning but also to measure the efficiency of the instruction for future reference.

Technology Knowledge (TK):
Quoting from the official website of Geometer's Sketchpad, which affirms my exposure to the software. The following are the features that are helpful for the current task. It equips students to:

- Generate algebraic explorations of arithmetic using models that support pattern, recognition, testing conjectures, and generalization and justification.
- Develop algebraic habits of mind as they drag objects or change parameters and observe results that reveal functional dependencies.
- Devise and test formulas using Sketchpad's calculator.
- Develop the concepts that are basis for our system of classifying shapes.
- Fully explore and understand the underlying principles of Euclidean constructions.
- Communicate their findings and explorations to their peers with easy-to-use presentation tools.

Specifically, with Geometer's Sketchpad, the task of visualizing the Pythagorean Theorem is possible.

II. TPACK Interplay

Pedagogical Content Knowledge (PCK)
The content concerns itself with understanding the Pythagorean Theorem in the absence of the rigid tenets of reasoning in formal proofs. By the definition of informal proof itself, "it is in less detail but more clarity" because it comes in high level sketches" that are usually time consuming with pencil and paper alone. Therefore, to picture the equation $c^2 = a^2 + b^2$, an avenue for visualization is imperative. In order to execute the instruction effectively, students start with a hard copy of the Pythagorean Square image to analyze before they start the informal proof where they can grow into establishing by drawing what the Pythagorean Theorem conveys. It is evident that the content itself calls for the specific pedagogy of visual-based learning.

Technological Content Knowledge (TCK)
In an article by Garofalo et. al. entitled *Promoting Appropriate Uses of Technology in Mathematics Teacher Preparation*, it exemplified that the use of Geometer's Sketchpad to prove the Pythagorean Theorem is a means that "address[es] worthwhile mathematics" [1]. Furthermore, the software itself advertises, it "visually demonstrate[s] the Pythagorean Theorem based on Euclid's proof". In light of this technology, which makes it possible to imagine the unimaginable, there is no doubt; it is a vital part of the instruction. The students will realize that no matter how they manipulate their digitally constructed triangle, it stays a right triangle and despite the changes on the side lengths when they are squared, the Pythagorean Theorem is satisfied. These scenarios can only be achieved with technology as the Geometer's Sketchpad.

Technological Pedagogical Knowledge (TPK)
Informal proof is tedious and time-consuming when illustrated manually with multiple tools such as the compass, straightedge, protractor, and the calculator for geometric construction: going back and forth with different things and calculating at the same time. There is neither a room nor an occasion to manipulate data or objects to observe various dimensions. With Geometer's Sketchpad, it has all the capacities of these tools with built-in calculator. It saves time and effort. It caters to adolescent thinking process with its capabilities for testing conjectures visually and experiencing the results in real time for countless possibilities. When the students create their triangle, Geometer's Sketchpad can measure the angles and side lengths by merely highlighting the parts with the click of a mouse. Once this is

accomplished, manipulating the vertices to move the shape around or to change its dimensions automatically populates new measurements. Hence, the efficacy of the teacher's instruction is enhanced while the depth of the students' understanding is increased.

Technological Pedagogical Content Knowledge (TPACK)

Taking from the preceding connections in PCK, TCK, and TPK, creating images of the equation affords students a realistic view of the numbers and variables in the theorem (PCK). Geometer's Sketchpad realizes this essential requirement of the content with its attributes that allow students to construct, measure, calculate, manipulate, and represent different perspectives to arrive at a reasonable generalization of the Pythagorean Theorem (TCK). The technology saves time and effort, accommodates the adolescent multi-perspective and relative thought processes, enhances the teacher's instruction, and increases student understanding (TPK). Put together, the content on informal proof of the Pythagorean Theorem that requires geometric construction is best implemented with visualization where students can create a mental picture of the theorem that is satisfied by Geometer's Sketchpad with its geometric sketch, explore, and computing abilities (TPACK).

[1]Garofalo, J., Drier, H., Harper, S., Timmerman, M.A., & Shockey, T. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. Contemporary Issues in Technology and Teacher Education, 1(1), 66-88.

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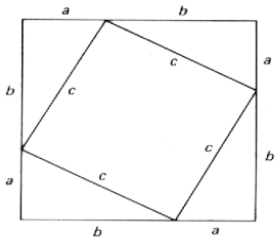
Lesson Plan 1 Addendums

The Pythagoras Square Worksheet Image

Name Grade Hour



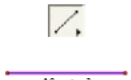
What do you see?

Write your observations about this image. Be prepared to share your ideas.

The R. O'Donnell Worksheet Images

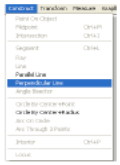
Using Sketchpad to Demonstrate the Pythagorean Theorem

1. Start Sketchpad if it isn't already running. If it is running, choose **New Sketch** from the **file** menu.
   A new, blank sketch window appears.
2. Choose the **Segment** tool from the Toolbox.
3. Construct a horizontal segment, in the middle of the blank screen, to be the bottom side of your triangle. Hold down the Shift key while doing this to help keep the segment perfectly horizontal.
   Your line can be as long as you choose, however, you don't want it to be extremely big, because you won't be able to see the whole triangle and the additional information that we'll add around it.
4. Click the **Selection Arrow**. The segment should already be highlighted, so all you need to do is also select the right endpoint. You'll know that you've selected the right number of objects if the Status Line says "Selected: 2 Objects."
   The Status Line is the informational text that appears in the bottom right (Mac) or bottom left (Windows) corner of the sketch window.
5. Your segment and its right endpoint should already be highlighted by clicking on them with the **Arrow**. Then, choose **Perpendicular Line** from the Construct menu.
6. Click on the **Point Tool** from the Toolbox. Your perpendicular line should still be highlighted from before, so use the Point Tool to construct a point above your segment on the perpendicular line.
   You will know if your point was constructed on the perpendicular line, because you will see that the line will be highlighted.
7. Using the **Selection Arrow**, click in the blank space to deselect all objects.
   It's important to deselect objects (by clicking in the blank space) before making new selections. Otherwise, you may end up selecting more objects than intended.



After step 3

Selected: 2 Objects



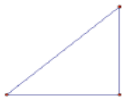
Ryan O'Donnell 1

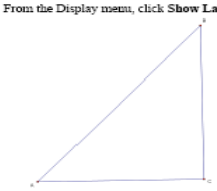
Using Sketchpad to demonstrate the Pythagorean Theorem

8. Still using the **Selection Arrow**, now highlight just the perpendicular line.
9. From the Display menu, click **Hide Perpendicular Line**.
   You should now have 3 points and one segment left on the sketchpad. If you hid the wrong objects, choose ***Undo Hide Objects*** *from the Edit menu and try again.*
10. Using the **Segment** tool, connect 2 segments to the 3 points to construct your triangle.

You should select any of the three points on the triangle and skew it to show that, no matter what you do, your triangle stays fixed as a right triangle. If your one vertex does not stay fixed at a 90° angle, you should start the tutorial over and try again.

11. Using the **Selection Arrow**, click in the blank space to deselect all objects.
12. Still on the **Selection Arrow**, select the three points on the triangle and, to correspond with mine, click, in order, the bottom left vertex, then the top point, and, finally, the bottom right.
13. From the Display menu, click **Show Labels**.





Ryan O'Donnell 2

Using Sketchpad to demonstrate the Pythagorean Theorem

14. Using the **Selection Arrow**, click in the blank space to deselect all objects.

15. With the **Selection Arrow**, click *(in order)* points A, B, then C and choose **Angle** from the Measure menu to show the measure of the angle selected in degrees.

16. Still using the **Selection Arrow**, click in the blank space to deselect all objects.

17. Follow Step 15, but you must first click the points B, C, and then A (*in order*), then choose **Angle** from the Measure Menu.

18. Still using the **Selection Arrow**, click in the blank space to deselect all objects.

19. Follow Step 15 again, but first click the points C, A, and then B, then choose **Angle** from the Measure Menu (*in order*).

20. Still using the **Selection Arrow**, click in the blank space to deselect all objects.

21. With the **Selection Arrow**, click the segments $\overline{AB}$, $\overline{CB}$, and $\overline{AC}$ (in no particular order) and select **Length** from the Measure menu.

You should select any points on the triangle and show it as much as you'd like to prove that even though the measures and two of the angles constantly change with your movement, the 90° angle always stays fixed.

Using Sketchpad to demonstrate the Pythagorean Theorem

22. Still using the **Selection Arrow**, click in the blank space to deselect all objects.

Assume that $\angle C$ is the right angle and all we know about the Pythagorean Theorem is that we must take the sum of the two legs, $\overline{AC}$ and $\overline{CB}$, in some way to have them equal the hypotenuse: $\overline{AB}$.
Note: They also all must share the same exponent.

23. In the Measure menu, select **Calculate.**
Note: We will use the Calculate option to demonstrate the formula for the Pythagorean Theorem.

24. Drag the mouse outside of the Calculate pop-up until $m\overline{AC}$ is highlighted, and then click.
$m\overline{AC}$ should show up in the white space.

25. Now, select the addition sign (+).

26. Repeat Step 24, except select $m\overline{CB}$ and click "OK."
A formula should show up with exact measurements. You may show the triangle again to show that the measurements will change along with the movement.

27. In the Measure menu, select **Calculate** again.

28. Follow Step 24, except, this time, select $m\overline{AB}$ (the hypotenuse) and click "OK."

Record your answers:

$m\overline{AC} + m\overline{CB}$ = _____

$m\overline{AB}$ = _____

Do the two values equal each other? _____

Using Sketchpad to demonstrate the Pythagorean Theorem

29. Repeat steps 23-28 a few more times, but continually raise your exponent by one in steps 24, 26, and 28. For example, you will try $m\overline{AC}\,\hat{}\,2 + m\overline{CB}\,\hat{}\,2 = m\overline{AB}\,\hat{}\,2$ and, then, $m\overline{AC}\,\hat{}\,3 + m\overline{CB}\,\hat{}\,3 = m\overline{AB}\,\hat{}\,3$ and so on.
Note: You must click the caret, followed by the exponent, after you enter your line segment. (Follow the picture.)

Record your answers:

$m\overline{AC}^2 + m\overline{CB}^2$ = _____

$m\overline{AB}^2$ = _____

Do these two values equal each other? _____

$m\overline{AC}^3 + m\overline{CB}^3$ = _____

$m\overline{AB}^3$ = _____

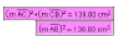
Do these two values equal each other? _____

$m\overline{AC}^4 + m\overline{CB}^4$ = _____

$m\overline{AB}^4$ = _____

Do these two values equal each other? _____

Which one of these formulas are equivalent?



_____

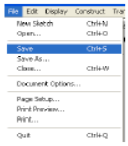
In the blank below, substitute the legs ($\overline{AC}$ and $\overline{CB}$) with *a* and *b*, respectively, and substitute the hypotenuse ($\overline{AB}$) with *c* to obtain the standard form of the theorem.

Pythagorean Theorem: _____
Voila! You now know the Pythagorean Theorem!

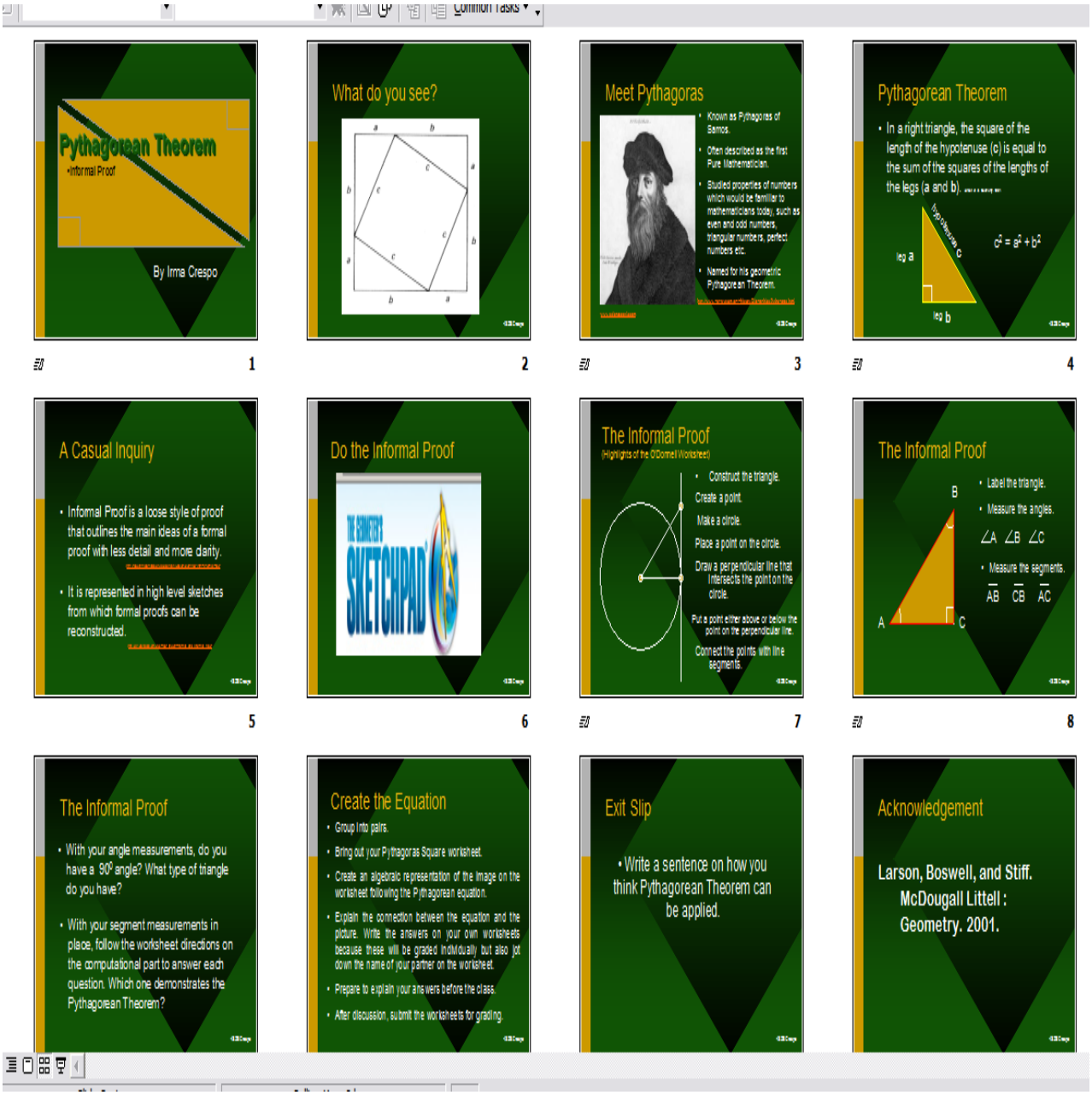
Using Sketchpad to demonstrate the Pythagorean Theorem

You should any points on the triangle and show it as much as you'd like, to demonstrate that, even though the measures of the two formulas are constantly changing with your movement, both measurement values stay the same.

30. Choose **Save** from the File menu. Give your sketch a descriptive name, such as Pythagorean, and click Save.



PowerPoint Presentation Images



Lesson Plan 2

Lesson Title: Applying Pythagorean Theorem

Lesson Author: Irma Crespo

Grade Level/Subject Area: Grade 11/Geometry

Time Alloted for Lesson: 45 minutes/One Class Meeting

Short Description of Lesson:
The students learn about the uses of Pythagorean Theorem on integers, finding measurements and areas using Pythagorean theorem with pen-and-paper method and with Geometer's Sketchpad.

Classroom Layout and Grouping of Students:
The set up varies from teacher centered on lectures and modeling, student to student with group work, student to class with presentations, and student workstations for individual practice within a group. Jigsaw with multiple ability grouping is the approach for this lesson plan.

Grade Level Content Expectations:

Michigan High School Content Expectations (HSCE):

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF
Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem in multi-step problems.

NETS•S (2007) **Performance Indicators for The Grade Level**

Profile for Technology (ICT) Literate Students
Grades 9–12 (Ages 14–18)

4. Employ curriculum-specific simulations to practice critical-thinking processes. (1.Creativity and Innovation and 4.Critical Thinking, Problem Solving, and Decision Making)

6. Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning, and career needs. (4.Critical Thinking, Problem Solving, and Decision Making, 5. Digital Citizenship, and 6. Technology Operations and Concepts)

Instructional Objectives

1. Knows the uses of Pythagorean Theorem. (Knowledge)

1.1 Defines Pythagorean Triple.
1.2 Identifies the use of Pythagorean Theorem in measurement problems.

2. Understands the uses of Pythagorean Theorem. (Comprehension)
   2.1 Interprets the use of Pythagorean Theorem in a word problem.
   2.2 Translates a verbal model into a Pythagorean Theorem model.

3. Applies the uses of Pythagorean Theorem. (Application)
   3.1 Computes Pythagorean Triples.
   3.2 Computes for the length of a hypotenuse with Pythagorean Theorem.
   3.3 Computes for the length of a leg with Pythagorean Theorem.
   3.4 Computes for the area with Pythagorean Theorem.

4. Analyzes the uses of Pythagorean Theorem with Geometer's Sketchpad. (Analysis)
   4.1 Illustrates Pythagorean Triples with Geometer's Sketchpad.
   4.2 Demonstrates finding side lengths using Pythagorean Theorem with Geometer's Sketchpad.
   4.3 Shows finding the area using Pythagorean Theorem with Geometer's Sketchpad.

5. Integrates the learning on the uses of Pythagorean Theorem. (Synthesis)
   5.1 Creates a Pythagorean Theorem problem.

Materials, Resources, and Technology: PowerPoint presentation, color-coded worksheets (red, yellow, blue and green), Geometer's Sketchpad, calculators, pens, paper, pencils, whiteboard, eraser, summary of student responses sheet

Students' Present Level of Performance and Skills: At this juncture, students should have already been exposed to axioms, postulates, properties, and theorems in addition to having made formal proofs at the beginning of the year. This is intended for students who have by now acquired experiences in geometric principles of reasoning and proof. Thus, it is offered in the last quarter of the year. The students, moreover, finished Algebra 1 and Algebra 2 in Grades 9 and 10, respectively. If digital technology exists, the students are already experienced with the application.

Instructional Procedures:

I. Anticipatory Set:
To prepare the students for the actual lesson on the uses of Pythagorean Theorem, the teacher lists the exit slip student responses from the previous meeting on how they think Pythagorean Theorem can be applied. (Found in Lesson 1 Supplemental Activity)
The teacher asks students randomly to read the components of the list one at a time and have them explain the connection between Pythagorean Theorem and the application.

II. Learning Activity:
The teacher opens up PowerPoint and indicates that today's lesson covers the uses of Pythagorean Theorem.

1. The teacher presents a lecture entitled Applying Pythagorean Theorem. Slides will expound and exemplify Pythagorean Triples, problems on finding side lengths and areas with Pythagorean Theorem.

2. Teacher-assigned Jigsaw grouping of multiple abilities follows. There are four members per group; each member is designated a color-coded worksheet with varying levels of difficulty [the students are not aware of the differentiation]. The red worksheet dwells on positive integers, the yellow worksheet delves into finding the length of a hypotenuse, the blue worksheet focuses on getting the measurement of a non-hypotenuse side, and the green worksheet emphasizes on an area problem.

3. At the beginning, the students with the same color worksheets converge to discuss resolutions to their delegated problem. Then, the students return to their appointed four-member grouping to teach their groups the processes on how they arrived at their answers.

4. After learning from each other, the assigned groups get together at a workstation and demonstrate the uses of Pythagorean Theorem with Geometer's Sketchpad – Pythagorean triples, side lengths, and areas.

5. The students are to save the Geometer's Sketchpad files on the desktop, print them out, attach same topic with color sheet, and staple all group worksheets together for submission.

III. Closure:
To sum up the hour's discussions, bring a conclusion to the lesson, and help students organize the learning that took place, they are to write one Pythagorean Theorem problem relating to its use. They can follow the format on the worksheets they worked on in the Jigsaw grouping. This is their exit slip.

Supplemental Activity: Extensions and Remediation:
For homework, each student is assigned to research one of the following:

- Perpendicular Postulate
- Geometric Mean Theorem
- Cross Product Property
- Addition Property of Equality
- Distributive Property
- Segment Addition Postulate
- Substitution Property of Equality

The paper has to state the postulate, property, or theorem. It has to include a proof, informal or formal. The paper is at least one page, typewritten on Word document, and with proper citation. (Recommended sites are the official website of Geometer's Sketchpad: keycurriculumpress.com, NCTM's Illuminations: illuminations.nctm.org, and Math Forum: mathforum.com)

Adaptations for Special Learners:
The Jigsaw grouping is particularly geared for multiple abilities. The color-coded worksheets reflect the varying difficulty of the tasks without the students being aware of the differences. In all cases, the students learn to solve the problems on their own and then, have the chance to teach what they learn to others in their group. Furthermore, the application starts with pen-and-paper followed by technology (Geometer's Sketchpad) so they are equipped with both ways of representing the problem regardless of their learning deficits. Hence, there is always a second chance to pick up on the concept if it didn't get

through the first time. Similar principle holds true with the individually assigned homework. Again, the students are not aware of the differentiation. It ranges from the easiest (Addition Property of Equality, which is simply known as the Commutative Property in elementary) to the most difficult (Perpendicular Postulate).

Assessment:

- Students will be assessed during class discussions for participation points recorded in the participation log. (Specific Objective 1: Knowledge)
- Students will be graded for the color-coded worksheets through the Jigsaw grouping for the pen-and-paper computations. (Specific Objectives1, 2, and 3 : Knowledge, Comprehension, Application)
- Students will be scored with a rubric for group work on the Jigsaw demonstrations via Geometer's Sketchpad. (Specific Objectives 2, 3, and 4: Comprehension, Application, and Analysis)
- Students will earn extra credit points towards a low-scoring project or quiz for completing the exit slip properly. (Specific Objective 5: Synthesis)

Student Products:
Anticipatory set: Participation Log
Learning Activity: Color-coded worksheets, Geometer's Sketchpad printouts
Closure: The exit slip on Pythagorean Theorem problem.

TPACK Analysis

I. TPACK Components

Content (C): Applying Pythagorean Theorem

Pedagogy (P): Jigsaw grouping, PowerPoint lecture

Technology (T): Geometer's Sketchpad

Content Knowledge (CK):
Pythagorean Theorem is one of the well-known theorems in geometry. Although it emanated from pure mathematics, today's perspectives extend its concept on applied mathematics. Thus, putting a scheme pristinely made for math pleasure (math for the sake of math) into functional use. There are more than one hundred proofs on Pythagorean Theorem, which accounts for its popularity. It is essential for students to learn it as a stepping-stone towards Trigonometry as well as an extension of their geometric skills. There are numerous resources available to study, learn, and teach Pythagorean Theorem. In this lesson plan, content is emphasized on the application of Pythagorean Theorem so students realize that there are uses for its equation on integers as well as on finding areas and measurements.

Pedagogy Knowledge (PK):
The students just finished learning Pythagorean Theorem by visualization. To relate its significance in geometry and in their lives, while taking into account the varying abilities in the classroom, the Jigsaw grouping is employed. The teacher-assigned members ensure the quality of the interaction because the

blending is not just on the basis of ability but also personality since the teacher has known the class from the beginning of the school year and this is a lesson that is implemented in the last quarter of the school year. Moreover, the lesson plan combines metacognition, pen-and-paper, and technology activities within the Jigsaw task. Again, to satisfy the non-linear growth of adolescents, the classroom activities involve various interfaces: teacher to student in PowerPoint lectures, student to teacher through inquiries, student to student in group tasks, interactivity between teacher, student, and technology in class discussions. Of course, these become routine structure the more they get familiar with this type of learning environment. In terms of evaluation, every task has an artifact assessed not only to gauge the quality of the students' learning but also to measure the efficiency of the instruction for future reference.

Technology Knowledge (TK):
Quoting from the official website of Geometer's Sketchpad, which affirms my exposure to the software. The following are the features that are helpful for the current task. It equips students to:

- Generate algebraic explorations of arithmetic using models that support pattern, recognition, testing conjectures, and generalization and justification.
- Develop algebraic habits of mind as they drag objects or change parameters and observe results that reveal functional dependencies.
- Devise and test formulas using Sketchpad's calculator.
- Develop the concepts that are basis for our system of classifying shapes.
- Fully explore and understand the underlying principles of Euclidean constructions.
- Communicate their findings and explorations to their peers with easy-to-use presentation tools.

II. TPACK Interplay

Pedagogical Content Knowledge (PCK)
The content focuses on applying Pythagorean Theorem, which requires both computation and analysis. The students in class possess varying learning abilities. Thus, Jigsaw grouping is the best-fit approach since it necessitates collaborative learning among students with different learning backgrounds. It opens a door of opportunity for students to work on tasks they are successful with while learning or re-learning concepts they need to improve on from their peers without the slight of labeling. In the same view, there is a mix of pen-and-paper method and technology use for the same concept in order to provide them time to reflect, redo, and eventually, illustrate the idea.

Technological Content Knowledge (TCK)
The mere fact that the uses of Pythagorean Theorem involve computation and analysis, already reveals the need to use Geometer's Sketchpad because of its capabilities to tackle algebraic and geometric applications. To reiterate, the software allows manipulation of numbers, variables, and objects. Students are able to create algebraic and geometric models, test formulas, and prove conjectures. In this lesson plan, all the students need to do is demonstrate an example of an application of Pythagorean Theorem. With the Pythagorean triples, the students can utilize Geometer's Sketchpad's formula calculator. While for areas and measurements, they can execute geometric constructions and set up their measurements. The software automatically computes the values while they manipulate the shape.

Technological Pedagogical Knowledge (TPK)
As opposed to plainly learning the pen-and-paper method where students only respond to the written requirements, Geometer's Sketchpad gives instant feedback of their keyed in information. Thus, there is an action/reaction process with the student/technology interface. On the Jigsaw grouping, the students

are able to digitally showcase to their group mates the steps they took to arrive at their answers and immediately prove to them that they solved the problems correctly.

Technological Pedagogical Content Knowledge (TPACK)

Taking from the preceding connections in PCK, TCK, and TPK, the content on the uses of Pythagorean Theorem entails computation and analysis. In order to fulfill equality of learning, the Jigsaw grouping designates tasks based on learning abilities and it also provides opportunities to represent solutions with both pen-and-paper method and technology (PCK). Geometer's Sketchpad realizes this essential requirement of the content with its attributes that allow students to construct, measure, calculate, manipulate, and represent different perspectives to arrive at a solution using Pythagorean Theorem (TCK). The technology accommodates different learning styles. It gives immediate feedback on student entries, which helps them recover immediately from the mistake and pursue the correct answers at a reasonable time. The students are able to clearly explain their steps and thus, develop their skills in mathematical discourse (TPK). Put together, the content on applying Pythagorean Theorem requires computation and analysis that is best implemented with Jigsaw grouping where students of multiple abilities learn together and teach each other with both traditional pen-and-paper and technological Geometer's Sketchpad for the demonstrative part of their solutions (TPACK).

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Lesson Plan 2 Addendums

Color-coded Worksheet Images

RED WORKSHEET

Calculate for a Pythagorean Triple

Given positive integers **m** and **n** such that **m** > **n**, find $2mn$, $m^2 - n^2$, $m^2 + n^2$.

1. **Write** down an **m** and an **n** such that **m** > **n**

m = _____

n = _____

Find:

2. $2mn$

3. $m^2 - n^2$

4. $m^2 + n^2$

Substitute the values calculated from #2, #3, and #4 on the Pythagorean Theorem: $c^2 = a^2 + b^2$

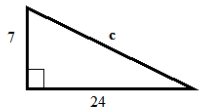
5.

What is your Pythagorean Triple?_____

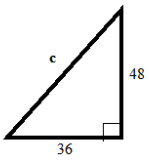
BLUE WORKSHEET

Find the length of the hypotenuse using Pythagorean Theorem.
Show your work.

1.



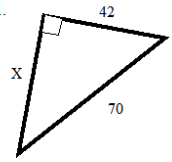
2.



YELLOW WORKSHEET

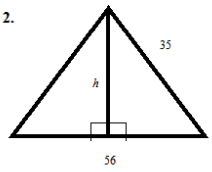
Find the length of the leg using Pythagorean Theorem.
Show your work.

1.



Find the length of the leg using Pythagorean Theorem and then find the area of the isosceles triangle. Show your work.

2.



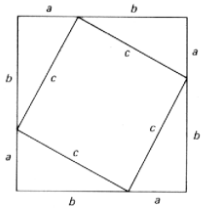
GREEN WORKSHEET

Area Formulas

Triangle
½ bh

Square
s^2

Given **b** = 3 m, **c** = 5 m.
Find the area of the whole square using Pythagorean Theorem.
Show your work.



PowerPoint Presentation Images

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Lesson Plan 3

Lesson Title: Formal Proof of Pythagorean Theorem

Lesson Author: Irma Crespo

Grade Level/Subject Area: Grade 11/Geometry

Time Alloted for Lesson: 45 minutes/One Class Meeting

Short Description of Lesson:
The students learn about the formal proof of Pythagorean Theorem using rules learned from the past: Perpendicular Postulate, Geometric Mean Theorem, Cross Product Property, Addition Property of Equality, Distributive Property, Segment Addition Postulate, and Substitution Property of Equality.

Classroom Layout and Grouping of Students:
The set up varies: teacher centered on lectures and modeling, student to student with group work, and student to class with presentations. Students are grouped based on the individual homework assigned, which is same ability grouping.

Grade Level Content Expectations:

Michigan High School Content Expectations (HSCE):

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF
Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem in multi-step problems.

NETS•S (2007) **Performance Indicators for The Grade Level**

Profile for Technology (ICT) Literate Students
Grades 9–12 (Ages 14–18)

6. Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning, and career needs. (4.Critical Thinking, Problem Solving, and Decision Making, 5. Digital Citizenship, and 6. Technology Operations and Concepts)

8. Model legal and ethical behaviors when using information and technology by properly selecting, acquiring, and citing resources. (3. Research and Information Fluency and 5.Digital Citizenship)

Instructional Objectives

1. Knows the rules pertaining to the formal proof of Pythagorean Theorem. (Knowledge)
   Defines Perpendicular Postulate.
   Defines Geometric Mean Theorem.
   Defines Addition Property of Equality.
   Defines Distributive Property.
   Defines Segment Addition Postulate.
   1.6 Defines Substitution Property of Equality.

2. Understands the concept of the rules pertaining to the formal proof of Pythagorean Theorem. (Comprehension)
   2.1 Explains Perpendicular Postulate proof based on Internet research.
   2.2 Explains Geometric Mean Theorem proof based on Internet research.
   2.3 Explains Addition Property of Equality proof based on Internet research.
   2.4 Explains Distributive Property proof based on Internet research.
   2.5 Explains Segment Addition Postulate proof based on Internet research.
   2.6 Explains Substitution Property of Equality proof based on Internet research.

3. Analyzes the concept of the rules pertaining to the formal proof of Pythagorean Theorem. (Analysis)
   3.1 Relates the rules to make the formal proof of Pythagorean Theorem.

4. Judges the credibility of a website and the validity of the mathematical proof from the website for the Pythagorean Theorem homework. (Evaluation)
   4.1 Appraises the credentials of the web source and the validity of the mathematical proof on a rule from that web source.

Materials, Resources, and Technology: PowerPoint presentation, student homework, Formal Proof worksheet, Jeopardy game, calculator, pens, paper, pencils, whiteboard, eraser

Students' Present Level of Performance and Skills: At this juncture, students should have already been exposed to axioms, postulates, properties, and theorems in addition to having made formal proofs at the beginning of the year. This is intended for students who have by now acquired experiences in geometric principles of reasoning and proof. Thus, it is offered in the last quarter of the year. The students, moreover, finished Algebra 1 and Algebra 2 in Grades 9 and 10, respectively. If digital technology exists, the students are already experienced with the application.

Instructional Procedures:

I. Anticipatory Set:

- To prepare the students for the main lesson, which is making a formal proof for Pythagorean Theorem, they are asked to bring out their individual homework. This assignment is a research on one of these rules: Perpendicular Postulate, Geometric Mean Theorem, Addition Property of Equality, Distributive Property, Segment Addition Postulate, and Substitution Property of Equality.

- The teacher groups the students into their respective topics. For each topic, the students are divided into at least three members or at most five members per group.

- The teacher explains the procedures of the activity and asks students if they have any questions.

- The students with the same topic discuss their individual findings and then, decide which of their researched proofs is best for class presentation. On a separate sheet of paper, the group writes or rewrites the proof on their assigned rule.

- When the students finish, the group proof has the names of the members on the paper.

- The groups present and elaborate on their proofs before the class. They take questions if there are any and answer them appropriately.

- When the presentations are done, every group submits the group work paper on top of the individual assignments for grading.

II. Learning Activity:
The teacher indicates that the Internet research and presentations they gave were meant to get them ready for making a proof on Pythagorean Theorem.

1. Since the class delved deeper into the rules by showing the proofs for each, the teacher sums up every rule with a PowerPoint lecture entitled, Review the Rules. It is a re-run on the key points - statements and examples.

2. The individual practice is next. The teacher distributes the Formal Proof worksheet and explains the directions.

3. The students work on the formal proof for Pythagorean Theorem.

4. After completing the Formal Proof worksheet, the students submit it for scoring.

III. Closure:
To end the unit on Pythagorean Theorem, the class celebrates with a Jeopardy Game.

The teacher divides the class into two groups: students on the left side of the room and students on the right side of the room.

The Jeopardy Game starts interactively with the PowerPoint. Two students from each group take turn to answer the questions. These are students who get their chance by consecutive order based on seating arrangement. The student who first answers the question correctly earns points for that group. The winning group attains the highest points and gets a reward.

Supplemental Activity: Extensions and Remediation:
For exit slip, the students write a sentence on what they learned about Pythagorean Theorem.

Adaptations for Special Learners:
The individually assigned homework is designed for multiple types of learners. It ranges from the easiest (Addition Property of Equality, which is simply known as the Commutative Property in elementary) to the most difficult (Perpendicular Postulate). Hence, when the grouping is implemented in the

anticipatory set, members of a group belong to the same or nearly the same ability band. Once more, nobody is aware of the differentiation.

The recapitulation of the rules on the PowerPoint target those who need more reinforcement and those who require a summation of all the segmental information presented in class.

Assessment:

1. Students will be assessed during class discussions for participation points recorded in the participation log. (Specific Objective 1: Knowledge)
2. Students will be graded with a rubric for the group presentations on their Internet researched proofs. (Specific Objectives 1 and 2: Knowledge and Comprehension)
3. Students will be scored for individual work on the formal proof of Pythagorean Theorem. (Specific Objectives 1, 2, and 3: Knowledge, Comprehension, and Analysis)
4. Students will be graded for the individual research homework. (Specific Objective 4: Evaluation[the highest order of cognitive learning])
5. Students will earn extra credit points towards a low-scoring quiz for completing the exit slip properly.

Student Products:
Anticipatory set: Group proof, Individual research (homework)
Learning Activity: Formal proof of Pythagorean Theorem
Supplemental or Extended Activity: The exit slip statement.

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TPACK Analysis

I. TPACK Components

Content (C): Formal Proof of Pythagorean Theorem

Pedagogy (P): Internet research

Technology (T): Internet

Content Knowledge (CK):
Pythagorean Theorem is one of the well-known theorems in geometry. Although it emanated from pure mathematics, today's perspectives extend its concept on applied mathematics. Thus, putting a scheme pristinely made for math pleasure (math for the sake of math) into functional use. There are more than one hundred proofs on Pythagorean Theorem, which accounts for its popularity. It is essential for students to learn it as a stepping-stone towards Trigonometry as well as an extension of their geometric skills. There are numerous resources available to study, learn, and teach Pythagorean Theorem. In this lesson plan, content is concentrated on its formal proof where students apply and analyze the rules that relate to proving Pythagorean Theorem.

Pedagogy Knowledge (PK):

At this phase, the students have studied and used the postulates, properties, and theorems needed to prove Pythagorean Theorem. To reconnect the students with this prior knowledge, they are asked to research on them not just on their definitions but more profoundly, on their proofs, which they are unaware of at this point. They have encountered these in the past to prove given conditions, not knowing how these rules themselves have proofs. The Internet research develops their investigative skills while applying previous geometric and reasoning experiences to judge the validity and accuracy of the mathematical information they are obtaining from the Internet.

The PowerPoint lecture is the summary of the major points of the rules. It helps students organize their thoughts. The Jeopardy Game reinforces knowledge in an engaging way – the students respond to questions where the instruction came through to them at least on the knowledge and comprehension levels.

As for the same ability grouping, students in the same level are able to relate their findings and are confident at presenting them to class. Thus, it builds on knowledge they already came across while keeping at par with the whole class instruction.

Technology Knowledge (TK):
The Internet has a wealth of information. It is a network of computers that communicate with each other in digital language. It is comparable to a library with infinite information at your fingertips. It is also a means to communicate with anybody, anywhere, anytime. The exchange of ideas is limitless. With the immensity of its scope, users are also wary of its drawbacks such as spam, malware, viruses, hackers, and identity thefts, to name a few. In education, Internet is meaningful when resources are credible and network communications are authenticated. In this lesson plan, the main concern is not navigational since the students are already using this technology outside school. The primary quest is how the students judge the credibility of the source and the validity of the proofs for their research.

II. TPACK Interplay

Pedagogical Content Knowledge (PCK)
The content concentrates on making a formal proof for the Pythagorean Theorem, which compels students to recall the relevant rules. To facilitate review of the rules, an Internet research is vital where students are able to use prior knowledge in picking the valid proofs from trustworthy sites. Given that the tasks are delegated based on ability, the students can make decisions with confidence since evaluating a proof requires a lot of exposure to determine its validity. With same ability grouping, students can relate mathematically with poise. The PowerPoint summation, meanwhile, helps students organize the pieces of information from the presentation into a generalized perspective. The Jeopardy Game emphasizes the concepts at the knowledge and comprehension stature.

Technological Content Knowledge (TCK)
The Internet is applicable technology towards understanding the rules that are used to prove Pythagorean Theorem. It helps students transition from recalling a known fact to assimilating its key ideas by inducing them to read, comprehend, and communicate the information online before making a pronouncement of its validity. It develops their reasoning skills as they are presented with numerous proofs from various websites, thereby, gaining countless views where prior experiences affect their choices for proofs relevant to the Pythagorean Theorem. Moreover, the Internet offers not only academic texts but also online discussions such as blogs or forums and even video vignettes to gather more data and acquire more ways to understand the content.

Technological Pedagogical Knowledge (TPK)

Internet research is the fastest and most convenient strategy in obtaining information for a next day presentation. The technology enhances the students' ability to connect the present with earlier knowledge. Applying the constructivist theory, by surfing the Internet and perusing through all the available sources, students accumulate invaluable knowledge that helps them make sense of new circumstances or conditions. In the pursuit of differentiation, the technology assists students in choosing information that are synchronized with their levels of understanding. It keeps them at ease with their mathematical thinking and discourse. Similarly, the summary imparted by the PowerPoint organizes all the concepts learned.

Technological Pedagogical Content Knowledge (TPACK)

Taking from the preceding connections in PCK, TCK, and TPK, producing a formal proof for the Pythagorean Theorem requires the application of rules learned in the past. To accomplish effective recall; research of the rules befitting the level of the student researcher, exchange of ideas with same ability peers, and impart a synopsis of gathered information increases meaningful recollection of facts needed for the content (PCK). The Internet technology completes the picture with readily available information, which offers students opportunities to read, comprehend, and communicate information in real time from web texts, online discussions, and electronic mails to set the foundation for making a formal proof (TCK). With Internet, students have a multitude of ways to acquire information that helps them build on their knowledge to make sense of new facts in a short time while accommodating the various learning styles and ability levels (TPK). Put together, the content on producing a formal proof of the Pythagorean Theorem requires significant recollection of rules previously learned, which is accomplished by research, same ability correspondence, and summation. In this regard, the Internet is the avenue to obtain the information in a jiff simultaneously with giving students options to choose the path that are parallel to their learning abilities and style. Thus, prepares them for the creation of a formal proof (TPACK).

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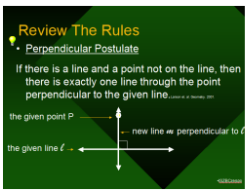
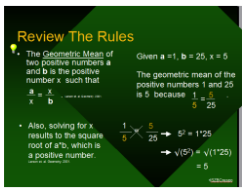
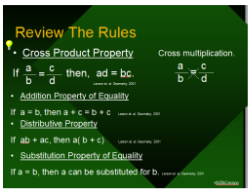
Lesson 3 Addendum

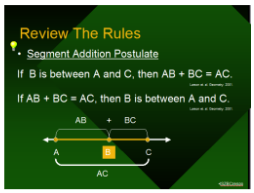
Formal Proof Worksheet Images

Formal Proof Worksheet







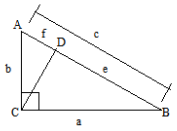


Directions:

1. Use Review the Rules to guide you with the formal proof.
2. Complete the following formal proof. The first line is already done for you.
3. Use all the rules reviewed.

In Δ ABC, ∠BCA is a right triangle.

Prove $a^2 + b^2 = c^2$



Plan for Proof:

Draw altitude line segment CD to the hypotenuse. Then apply Geometric Mean Theorem.

Statements	Reasons
1. Draw a perpendicular from C to line segment AB.	Perpendicular Postulate

Adapted from Larson et.al. Geometry. 2001. Illinois: McDougall Litell p. 535

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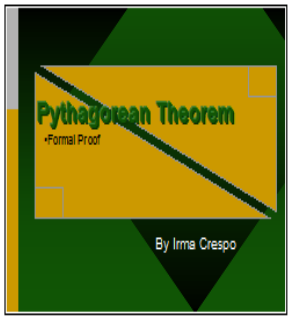
PowerPoint Presentation Images



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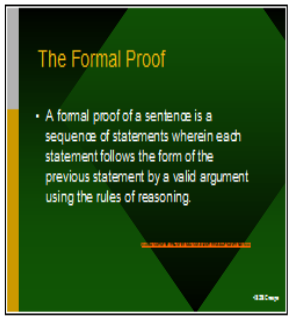

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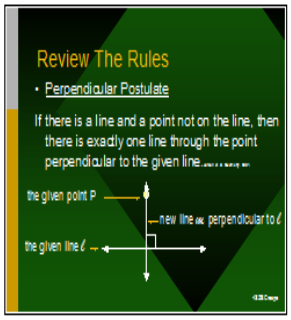

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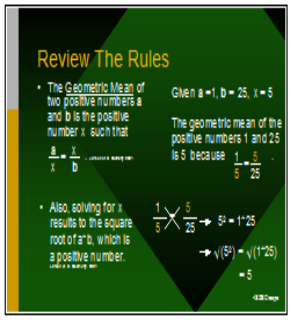

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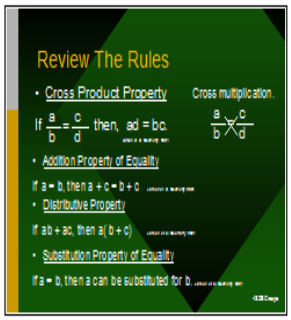

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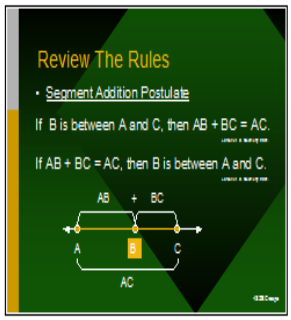

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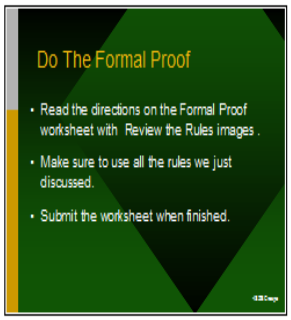

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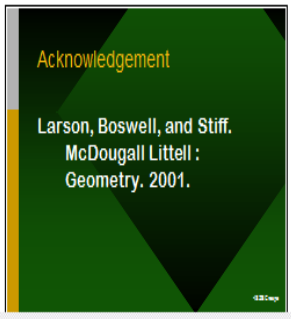

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